Chapter 1

Syllogistic

1. Before studying Leibniz's original contributions to logic, it is worth reviewing what he thought of the traditional logic of Aristotle and the scholastics, and in particular his views on the theory of the syllogism. For the latter he always professed a profound admiration:

> I hold the invention of the syllogistic form to be one of the most beautiful inventions of the human mind, and indeed one of the most notable. It is a sort of universal mathematics,[1] the importance of which is too little known. One can say that a type of infallibility is contained in it, provided that one knows how to use it well and has the opportunity to do so, something that is not always the case.[2]

The best proof of the value Leibniz saw in formal logic, and especially in the syllogism, is his successful employment of it in his controversy on dynamics with Denis Papin.[3] They pushed the argumentation all the way to the twelve or thirteenth syllogism, after which they came to agreement, while beforehand they had not succeeded in understanding one another.[4]

Yet this veneration of logical form and his admiration for the theory of the syllogism did not prevent Leibniz from regarding Aristotle's work as imperfect or from wanting to amend and complete it, following the example of the scholastics themselves. Toward the end of his life he wrote these lines, which sum up his own theory:

> The logic of syllogisms is truly demonstrative, just like arithmetic or geometry. In my youth I demonstrated not only that there are four figures, which is easy,[5] but also that each figure has six useful moods, and could

[1] Note this expression, which is repeated later by Philalethes (§9) and which we will later come to understand better (see Chap. 2, §1, and Chap. 7, §14).

[2] *New Essays*, IV.xvii.4 (1704). Cf. Leibniz to Koch, 15 July 1715 (*Phil*., VII, 481); Leibniz to Des Bosses, 2 May 1710 (*Phil*., II, 402); Leibniz to Lange, 5 June 1716 (Note XIX); Leibniz to Eler, 10 May 1716 (Note XVIII).

[3] *Response to Reasonings set forth by D.P. last January in these Acta Concerning the Laws of Nature and the True Appraisal of Motive Force, against the Cartesians* (*Acta Eruditorum*, 1691; *Math*., VI, 204-11). Several documents relating to this debate can be found in Leibniz's manuscripts: *Summary of the Formal Dispute I Had with Mr. Papin on the Estimation of Force, from the First to the Thirteenth Syllogism* (LH IV 2, 8 a; unpublished); LH IV 2, 8 b: the piece published by Gerhardt as an appendix to the article from the *Acta* (*Math*., VI, 211-5); LH XXXV 9, 7: *Synopsis of the Controversy on the True Estimation of Motive Force, Excerpted by Denis Papin*, in Papin's hand with this note by Leibniz: "In this synopsis, the state of the controversy and the reasons put forth on both sides do not seem to me to have been set out accurately enough" (Bodemann, 301).

[4] Leibniz to Gabriel Wagner, 1696 (*Phil*., VII, 522).

[5] See Appendix I.

> have neither more nor fewer, whereas ordinarily one allots but four to the first and second, and five to the fourth.[6]

2. The demonstration to which Leibniz was thus referring is found in his *Dissertation on the Art of Combinations*,[7] composed at the age of nineteen in 1666. From the beginning to the end of his career, his opinion on this subject did not change. Among the applications of the art of combinations, Leibniz cites the determination of the number of moods of the categorical syllogism.[8] In this, he follows the example of Johannes Hospinianus, who had published in Basel in 1560 a small work in which he found a total of 512 moods, 36 of which were valid.[9] As was his habit, Leibniz set out to discover Hospinianus's conclusions for himself, using his combinatorial art to construct in a systematic way all possible moods.

For each proposition he distinguishes, following Hospinianus, four quantities: universal (U), particular (P), indefinite (I), and singular (S); and two qualities: affirmative (A) and negative (N). He then combines the quantities and qualities separately, each taken three at a time (the first letter representing the major premise, the second the minor and the third the conclusion); each combination thus represents a different mood, either with respect to quantity or with respect to quality. In this way he finds 64 (4^3) different combinations in quantity, of which only 32 are valid. The others he eliminates by means of two classical rules: "from particulars alone nothing follows" and "the conclusion cannot exced the premises in quantity."[10] Similarly, he finds eight (2^3) different combinations in quality, of which only three are valid, namely: AAA, NAN, ANN. The others are excluded by the classical rules "from negatives alone nothing follows" and "the conclusion follows the lesser part in quality."[11] He then combines the 32 quantitative moods with the three qualitative moods, so as to form 96 (32 x 3) distinct moods.[12] From these 96 moods he subtracts the eight that are contained in the type *Frisesmo* and which belong to no figure.[13] There remain for him 88 useful, that is to say, valid moods.

3. This proliferation of valid moods clearly derives from the fact that Hospinianus recognized four different quantities in place of the two quantities (U and P) distinguished by Aristotle and the scholastics. As a result, we should find several valid moods which are merely instances or particular cases of a single traditional mood. To the single mood *Darii*, for example, there correspond nine different combinations (in quantity): UII, USS,

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6 Leibniz to Bourguet, 22 March 1714 (*Phil*., III, 569). Cf. Leibniz to Koch, 2 September 1708: "I once discovered that there are six valid moods in each figure, and can be neither more nor fewer" (*Phil*.,VII, 478); and Leibniz to G. Wagner, 1696 (*Phil*.,VII, 519).

7 *Phil*., IV, 27-104; *Math*., V, 7-79.

8 Application VI (*Phil*., IV, 46; *Math*., V, 23).

9 Entitled *Non esse tantum 36 bonos malosque categorici syllogismi modos, ut Aristoteles cum interpretibus docuisse videtur, sed 512, quorum quidem probentur 36, reliqui omnes rejiciantur*. In *Catalog of Discoveries in Logic* (LH IV 7B, 4 Bl. 32), Leibniz cites Hospinianus as having enumerated the absolute moods of the syllogism.

10 These are rules VIII and VII (2) of Appendix I.

11 These are rules VI and VII (1) of Appendix I, to which it is necessary to add rule V ("two affirmative premises cannot give rise to a negative conclusion").

12 If the 64 quantitative moods are combined with the 8 qualitative moods (one by one), we get the 512 (64 x 8) moods envisioned by Hospinianus.

13 Later we shall see why this is so (§4).

UPP, UIS, USI, UIP, UPI, USP, UPS. Moreover, Hospinianus recognized as distinct moods the *subalternate* moods, i.e., moods in which the conclusion derives by subalternation from the conclusion of the universal moods: *Barbari* (derived from *Barbara*), *Celaro* (from *Celarent*), *Cesaro* (from *Cesare*), and *Camestros* (from *Camestres*).

Leibniz approves of these various reforms,[14] but accuses Hospinianus (with reason) of having assimilated singular propositions to particular ones, and he shows that, on the contrary, they are equivalent to universal propositions, since the subject is to be taken in the totality of its extension.[15] Similarly, indefinite propositions must be assimilated to particular ones, with the result that Leibniz finally recognizes no more than the two classical quantities (U and P). Consequently, the 32 combinations in quantity reduce to the following four: UUU, UUP, UPP, PUP, each of which gives rise to seven others when U is replaced with S or P with I. Each one of these moods then represents eight of Hospinianus's moods. These four quantitative moods taken one at a time with the three qualitative moods (AAA, ANN, NAN) produce the 12 *general simple moods* tabulated below.

4. These 12 simple moods give rise to 24 *figured moods* when we take into account the diversity of figures, i.e., the place of the middle term in each of the premises. Here Leibniz energetically defends the fourth figure against its critics (following the example of Hobbes) and proves that it is as legitimate as the others.[16] In this syllogism, for example,

Every animal is a substance
Every man is an animal (therefore)
Some substance is a man

the minor term is "substance," so the minor premise is the first of the two premises and the major is the second. The middle term ("animal") is the predicate of the major and the subject of the minor, which is characteristic of the fourth figure.[17] Leibniz further notes that, just as the second figure arises from the first by conversion of the major premise, and the third by conversion of the minor, so the fourth arises from the first figure by conversion of the conclusion (which entails the transposition of the premises). Finally, he recalls the rules specific to each figure: (1) In the first and second, the major is always universal; (2) In the first and third, the minor is always affirmative; (3) In the second, the conclusion is always negative and in the third it is always particular; (4) In the fourth, the conclusion is never universal affirmative, the major is never particular negative, and, if

[14] *Phil*., IV, 50; *Math*., V, 27.

[15] *Phil*., IV, 48; *Math*., V, 25-26.

[16] "The fourth figure is just as good as the first" (*Phil*., IV, 52; *Math*., V, 29). Cf. the letter to Bourguet cited above (§1).

[17] Cf. the letter to Koch of 2 September 1708 (quoted in Appendix I), which shows that Leibniz did not change his opinion on this point. We may add that, although he considers the order of the premises to be irrelevant in the determination of figures, he nonetheless preferred the order adopted by Ramus and Gassendi, in which the minor comes before the major, as in the syllogism above; he notes that this order corresponds to the relation of *predication*, while that of Aristotle answers to the consideration of *comprehension* ("B is in A" instead of "All A is B"). Cf. *New Essays*, IV.xvii.8, and 4 (quoted Chap. 2, §1).

the minor is negative, the major is universal affirmative.[18] From these rules it follows that each simple mood is not valid in every figure, and that the 12 simple moods give rise only to the 24 figured moods contained in the following table:[19]

Simple Moods	First Figure	Second Figure	Third Figure	Fourth Figure
UA, UA, UA	Barbara	—	—	—
UN, UA, UN	Celarent	Cesare	—	—
UA, UN, UN	—	Camestres	—	Calerent
UA, UA, PA	Barbari	—	Darapti	Baralip
UN, UA, PN	Celaro	Cesaro	Felapton	Celanto
UA, UN, PN	—	Camestros	—	Fapesmo
UA, PA, PA	Darii	—	Datisi	—
UN, PA, PN	Ferio	Festino	Ferison	Fresison
UA, PN, PN	—	Baroco	—	—
PA, UA, PA	—	—	Disamis	Ditabis
PN, UA, PN	—	—	Bocardo	—
PA, UN, PN	—	—	—	—

Only the last simple mood (IEO) gives rise to no figured mood at all. It has no place in any of the first three figures, since its major premise is particular and its minor negative, and it violates the last rule of the fourth figure. It belongs therefore to no figure.[20]

[18] For the justification of these rules, see Appendix I.

[19] We have corrected this table in accordance with the instructions given by Leibniz himself in the note he published in 1691 in the *Acta Eruditorum* to protest the unauthorized reprinting of his dissertation and to correct certain errors in it (*Phil*., IV, 103-104). He observed that there is no OAO mood in the fourth figure, though there is an AEE mood, and concluded: "Thus, instead of putting *Colanto* next to *Bocardo*, *Calerent* ought to be put next to *Camestres*." In fact, one finds *Colanto* listed among the moods of the fourth figure (*Phil*., IV, 52; *Math*., V, 30). Gerhardt wanted to make this correction, but misunderstood it: he entered *Calerent* beneath *Ditabis* and *Camestres* beneath *Disamis* (*Phil*., IV, 53; *Math*., V, 31). But Leibniz did not mean that in replacing *Colanto* by *Calerent* it was also necessary to replace *Bocardo* by *Camestres* (which is of the second figure and is thus wrongly repeated); rather he meant that in place of *Colanto* on *Bocardo*'s line one should put *Calerent* on *Camestres*' line. This is what we have done, returning *Bocardo* to its rightful place. Leibniz also corrected another error which, having been introduced in this new edition, passed into Gerhardt's: "*Frisesmo* ought to be put under figure 0 (or the null figure) and not under IV (the fourth). Nor indeed can the mood IEO be assigned to any figure if it has PA as its major, UN as minor, and PN as conclusion." We have chosen to omit the column 0, since it answers to no figure, as well as the name of this false mood *Frisesmo* (the cause of the preceding error). In a letter to Placcius (16 November 1686), Leibniz was led to say, in regard to Hospinianus: "If I may add one more thing, I was myself mistaken in my small book *On the Art of Combinations* when I gave the number of useful moods, for the moods of the fourth figure ought to be AEE, AAI, EAO, EIO, AEO" (*Dutens*, VI.1, 31-2). Here he forgot the mood *Ditabis* (unless this was the fault of the editor or the printer).

[20] This demonstration seems simpler and more convincing than the one Leibniz gives by example, for, as he remarks, examples can be conclusive only in virtue of their content and not in virtue of their form (*Phil*., IV, 54; *Math*., V, 31-32). It is true that in this connection he gives some extremely ingenious rules for finding infallible examples, where the subject matter cannot conceal the invalidity of the form. It is to these rules that he makes allusion in the *New Essays*, IV.xvii.4: "One ordinarily makes use of examples in order to justify one's inferences, but this is not always a very trustworthy procedure, even though there *is* a way of choosing examples that would not be true unless the inference were valid."

Leibniz thus finds six valid moods in each of the four figures. This symmetry pleases him and seems to him a mark of truth, for, as he says, nature is regular in all things, and he finds this order to be every bit as noteworthy as the number of regular solids (polyhedra).[21] As he observes elsewhere,[22] it is obtained by simply adjoining to the universal moods in each figure the corresponding subalternate moods, as their names already sufficiently indicate: *Barbari*, *Celaro*, *Cesaro*, *Camestros*. In the fourth figure, however, this correlation is masked by a poor choice of name: the subalternate mood of *Calerent* (which we call *Camenes*) is the mood called *Fapesmo* (which we should call *Camenos*), while the mood *Celanto* is our *Fesapo* (or *Fespamo*).[23]

5. To reduce the moods of the last three figures to those of the first, in accordance with the Aristotelian tradition, Leibniz adopted an ingenious method which had been suggested to him by his teacher Jacob Thomasius but whose discovery goes back to Ramus.[24] This method is the one Leibniz calls *regression* and which today we call *reductio ad absurdum*. It consists in taking as premises of a new syllogism one of the premises of the given syllogism together with the negation of its conclusion, and in deducing from these the negation of the other premise. This is indeed the process of proof by absurdity: if a syllogism is valid, its conclusion can be false only if at least one of the premises is false; if we assume then that one of the two premises is true, the other must be false.[25] Leibniz preferred this method to the classical method of reduction (through conversion) because it rests on the principle of contradiction[26] and because it has the advantage of invoking the least number of principles. It is true that this method is applicable only to moods of the second and third figures, while those of the fourth figure must in addition undergo conversion to be reduced to the first, but “it is quite significant that the very conversions needed are demonstrated by the second or third figure, which are themselves demonstrable independently of any conversions,” that is, “by the principle of contradiction alone.”[27]

In sum, once the four principal moods of the first figure have been established, one can demonstrate subalternation by means of *Darii* and *Ferio*, and then the secondary moods *Barbari* and *Celaro* by means of subalternation. From the six moods of the first

[21] Leibniz to Gabriel Wagner, 1696 (*Phil*., VII, 518).

[22] *New Essays*, IV.xvii.4.

[23] Towards the end of his life, Leibniz seems to have sought other names for the new moods, no doubt with an eye to indicating better their independence from the ancient moods: in the first figure, *Gabali* and *Legano*; in the second, *Gaceno* and *Lesaro*; in the fourth, *Cademop* (LH VI 15).

[24] *Phil*., IV, 55; *Math*., V, 33. Cf. *New Essays*, IV.ii.1: “In my youth when I was examining these things in detail, I noticed that all the moods in the second and third figures can be derived from those of the first by this method alone.” Cf. LH IV 6 Bl. 15; LH IV 7B, 4 Bl. 7. For his views on Ramus as a logician, see *Phil*., VII, 67. In *Catalog of Discoveries in Logic,* we read: “Peter Ramus [discovered] a demonstration of conversion using identities and the rules governing the figures, or a demonstration that the other figures arise from the first by means of identities and conversions” (LH IV 7B, 4 Bl. 32).

[25] Cf. Appendix I, §9 and the fragment entitled *Logical Difficulties* (*Phil*., VII, 212). This kind of verification of syllogisms by regression is clearly defined in an unpublished fragment likely dating from 1679 (LH IV 7B, 2 Bl. 15 verso).

[26] Though less immediately than Leibniz thinks.

[27] *New Essays*, IV.ii.1. Leibniz concludes from this that the fourth figure “is slightly more distant from the first than the second or third, which are on a par and equally distant from the first; whereas the fourth still requires the second and third for its demonstration” (ibid.). This conclusion does not accord at all well with his opinion on the legitimacy of the fourth figure.

figure, one then deduces the twelve of the second and third by regression. Next, one demonstrates the laws of conversion using the syllogisms of the second and third figures and finally deduces the moods of the fourth figure through conversion.[28]

6. Let us first see how subalternations and conversions can be demonstrated using syllogisms, taking as one premise an identical proposition "All A is A" or "Some A is A."[29]

The subalternation of an affirmative proposition is demonstrated by means of *Darii*:

All A is B
Some A is A (therefore)
Some A is B.

The subalternation of a negative proposition is demonstrated by means of *Ferio*:

No A is B
Some A is A (therefore)
Some A is not B.[30]

Similarly, a universal affirmative can be converted by means of *Darapti*, from the third figure:

All A is B
All A is A (therefore)
Some B is A.[31]

The universal negative is converted by means of *Cesare*, from the second figure:

No A is B
All B is B (therefore)
No B is A.

Finally, the particular affirmative is converted by means of *Datisi*, from the third figure:

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[28] *On the Mathematical Determination of Syllogistic Forms* (LH IV 7C Bl. 83-84). Cf. *Principles of a Rational Calculus* (LH IV 7B, 2 Bl. 1) and *Logical Definitions*: "It is worth noting that subalternation, just like conversion, can be demonstrated with the aid of syllogisms" (*Phil*., VII, 209).

[29] "Conversion can be demonstrated through a syllogism, by adding an identical proposition." *On the Art of Combinations* (*Phil*., IV, 55; *Math*., V, 33).

[30] It must be noted that subalternation is as valid as the so-called identical premise "Some A is A." While the universal "All A is B" has no *existential import*, the particular "Some A is B" does, in that it implies that there are some As, for "some A" is considered to be the contradictory of "no A." As a result, to say "Some A is A" is to say that there exist As, and this is in no way an identical proposition. The particular statement therefore contains something more than the corresponding universal, namely this claim of existence, and cannot be deduced from it. Modern logic, accordingly, does not permit subalternation.

[31] *Phil*., IV, 55; *Math*., V, 33. It is clear that this *partial* conversion has the same validity as the mood *Darapti*, which modern logic regards as invalid. Specifically, the particular conclusion has an existential import that is contained in neither of the universal premises and which cannot therefore be deduced from them.

All A is A
Some A is B (therefore)
Some B is A.[32]

Leibniz even tries to convert the *particular negative* in this same way using *Baroco*. As he says:

All A is A
Some A is not B (therefore)
Some B is not A.

But here he is mistaken; this is not a syllogism of the mood *Baroco* (of the second figure). It is a syllogism of the third figure, where there is no AOO mood, and so has the mood OAO (*Bocardo*). Nor can this conversion be demonstrated, as he thought, by a syllogism in *Colanto*, since this mood does not exist in the fourth figure, as he later recognized.[33] In short, the above syllogism is simply invalid, as is the conversion that he claims to demonstrate from it.

The only valid conversion from a particular negative is the one Leibniz mentions soon thereafter. The proposition "Some man is not wise" is equivalent to the particular affirmative "Some man is non-wise," which can be simply converted into "Some non-wise thing is a man."

But Leibniz thinks that he should reject indefinite forms of propositions, i.e., forms in which a term is negative, which had been allowed by Johann Christoph Sturm.[34] He rightly contends that this distinction applies to the content of a judgment and not to its form, and therefore cannot serve as a basis for new syllogistic moods. Similarly, he refutes the discovery of a mood *Daropti* which had been proposed by his teacher Thomasius as a means of converting the particular negative:

Every man is a man
Some man is not wise (therefore)
Some non-wise thing is a man.

He notes that the minor premise is actually "Some man is non-wise," which is similar to the conclusion and which, as he says, entails that this syllogism is in the mood *Datisi*, not *Baroco*.

On the other hand, he elsewhere reduces negative propositions to an affirmative form by giving them an *indefinite* predicate, that is, by making the negation affect the predicate. From this he establishes the falsity of the classical axiom according to which two negative premises give rise to no conclusion. Consider the two negative premises:

No man is a stone,
No man is an angel.

[32] *Phil*., IV, 55-56; *Math*., V, 33. Cf. *New Essays*, IV.ii.1.
[33] In his note in the *Acta Eruditorum* of 1691 (*Phil*., IV, 104).
[34] *Compendium Universalium seu Metaphysicae Euclideae* (The Hague: Adrien Vlacq, 1660).

These can be transformed into two affirmatives:

Every man is a non-stone
Every man is a non-angel

from which follows (by *Darapti*):

Some non-angel is a non-stone.

This syllogism has only three terms (*man*, *non-angel*, *non-stone*), unlike those obtained in applying this same transformation to the negative moods of the second figure. The middle term occurs as a predicate twice in these latter moods and thus is split into two terms, one positive and one negative.[35]

All the preceding deductions have the goal both of showing the utility of identical propositions in reasoning and of defending them against the charge of insignificance and sterility made by empiricist logicians.[36]

7. We can now present the deduction of the 24 valid moods by means of regression or *reductio ad absurdum*. First we need to demonstrate the four moods of the first figure: *Barbara*, *Celarent*, *Darii*, and *Ferio*. Leibniz deduces them from a single principle which he calls the "foundation of the syllogism" and which is equivalent to Aristotle's "dictum of all and none." In two unpublished studies, this principle is formulated extensionally as follows: "If the middle term is included in the major, or is excluded from it, then any minor term that is contained in the middle is similarly included in, or excluded from, the major."[37] From this principle we can immediately deduce the two affirmative moods *Barbara* and *Darii*, on the one hand, and the two negative moods *Celarent* and *Ferio*, on the other. And we have already seen how we can deduce subalternation from these moods and how we can get the secondary moods *Barbari* and *Celaro* through subalternation.[38]

With this done, the six moods of the first figure give rise, through regression, to the same number of moods in the second and third figures. If we take as premises the major of each of them, together with the negation of its conclusion, we obtain as conclusion the

[35] See the unpublished opuscule *On the Verification of Logical Form by the Drawing of Lines* (LH IV 7B, 4 Bl. 10 verso). This reduction is outlined in the *Calculus of Consequences* of April 1679 (LH IV 5, 8 Bl. 24-27).

[36] "Identities also have their applications, and it will soon be clear that, however insignificant it may appear, no truth is wholly unproductive, and that the foundations of other truths are already contained in these laws" (*Phil*., VII, 300). Cf. *New Essays*, IV.ii.1.

[37] "The fundamental syllogistic law is this: If all of something C falls inside of something D, or if all of C falls outside of D, then whatever is in C falls inside of D, in the first case, and outside of D, in the second. This is what is generally called the dictum of all and none." *On Mathematical Definition* (LH IV 7C Bl. 83). Elsewhere Leibniz formulates the following two axioms: "1. Whatever is contained in something nonexistent is itself nonexistent.... 2. Whatever is contained in something excluded is itself excluded," which he illustrates with linear schemata (LH IV 7B, 3 Bl. 11).

[38] Cf. *New Essays*, IV.xvii.4: "So the two additional moods of the first figure [*Barbari*, *Celaro*] can be demonstrated from the first two ordinary moods of that figure [*Barbara*, *Celarent*] through the intervention of subalternation, which itself is demonstrable using the other two moods of the same figure [*Darii*, *Ferio*]."

negation of its minor. In this way we get the six moods of the second figure. If we then take as premises the minor and the negation of the conclusion, we get the negation of the major as conclusion. In this way we find the six moods of the third figure. The set of these 12 regressions is clearly represented in the following table:[39]

Barbara	ACD	ABC	ABD	*Barbara*	ACD	ABC	ABD
Regression	ACD		OBD	Regression		ABC	OBD
Hence		OBC		Hence	OCD		
Baroco	ACD	OBD	OBC	*Bocardo*	OBD	ABC	OCD
Celarent	ECD	ABC	EBD	*Celarent*	ECD	ABC	EBD
Regression	ECD		IBD	Regression		ABC	IBD
Hence		OBC		Hence	ICD		
Festino	EDC	IBD	OBC	*Disamis*	IBD	ABC	ICD
Darii	ACD	IBC	IBD	*Darii*	ACD	IBC	IBD
Regression	ACD		EBD	Regression		IBC	EBD
Hence		EBC		Hence	OCD		
Camestres	ACD	EBD	EBC	*Ferison*	EBD	IBC	OCD
Ferio	ECD	IBC	OBD	*Ferio*	ECD	IBC	OBD
Regression	ECD		ABD	Regression		IBC	ABD
Hence		EBC		Hence	ICD		
Cesare	ECD	ABD	EBC	*Datisi*	ABD	IBC	ICD
Barbari	ACD	ABC	IBD	*Barbari*	ACD	ABC	IBD
Regression	ACD		EBD	Regression		ABC	EBD
Hence		OBC		Hence	OCD		
Camestros	ACD	EBD	OBC	*Felapton*	EBD	ABC	OCD
Celaro	ECD	ABC	OBD	*Celaro*	ECD	ABC	OBD
Regression	ECD		ABD	Regression		ABC	ABD
Hence		OBC		Hence	ICD		
Cesaro	ECD	ABD	OBC	*Darapti*	ABD	ABC	ICD

As to the moods of the fourth figure, Leibniz undoubtedly demonstrated these following the classical method, reducing them to the moods of the other figures by means of conversions.[40]

8. We have just seen how Leibniz grounds the principles of the syllogism extensionally. There are, however, other studies in which, on the contrary, he relies on a consideration of the comprehension of concepts. He then defines relations of inclusion between the terms of a proposition as follows: “A includes B” signifies that B is universally affirmed of A, that is, All A is B. Similarly, “A excludes B” signifies that B is universally denied

[39] *On Mathematical Definition* (LH IV 7C Bl. 84). Leibniz designates the minor, the middle, and the major terms by the consonants B, C, and D, respectively. He represents each proposition by three letters, the first being the vowel which indicates its quantity and quality, the second the consonant designating the subject, and the third the consonant which designates the predicate. It will be noticed that the moods of the second figure are found in the left-hand column, and those of the third in the right-hand column.

[40] See the rules for this reduction in Appendix I, §8.

of A, that is, No A is B.[41] We see that he conceives these relations of inclusion and exclusion from the point of view of the comprehension of concepts. It is important to note, though, that from this point of view, one can define (at least directly) only universal propositions, since concepts include or exclude each other in their entirety, and it would be absurd to suppose a *partial* inclusion or exclusion between them.

As a result, particular propositions can be defined only as negations of the universal propositions of which they are contradictories (namely, O as the negation of A and I as the negation of E).[42] It follows from this that the only primitive moods are the five universal moods (*Barbara*, *Celarent*, *Cesare*, *Camestres*, *Camenes*). Since their conclusions are universal, so too are their premises, and they are, as a result, uniquely composed of universal propositions that can be interpreted in terms of comprehension as affirming total inclusions or exclusions among their terms.

9. To these five universal moods Leibniz joins the five particular moods derived from them through subalternation of the conclusion (the premises remaining the same), namely *Barbari*, *Celaront*, *Cesaro*, *Camestros*, *Camenos*.[43] He then transforms these ten moods with universal premises by regression: each one gives rise to *two* other moods, by joining to either its major or its minor premise the negation of its conclusion and deducing from them the negation of the other premise. In this way we get 20 derived moods, and hence 30 moods in all. But these must be reduced to the 24 valid moods, so six of them must be duplicates. This is indeed what we find in carrying out the double reduction indicated by Leibniz.[44]

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[41] *Phil.*, VII, 208.

[42] "A particular affirmative is a denial of exclusion, and a particular negative is a denial of inclusion" (LH IV 7B, 4 Bl. 26). Cf. *Phil.*, VII, 208, nos. 3 and 4; LH IV 7B, 4 Bl. 3 recto. In other places Leibniz defines negative propositions as negations of affirmative ones, which comes to the same thing (*Elements of a Calculus*, §8, April 1679, LH IV 5, 8b; *General Investigations Concerning the Analysis of Concepts and Truths*, §§120-121, 1686, LH IV 7C Bl. 28 recto). Of course one can also define particular propositions as *subalternates* of universal propositions, and this is what Leibniz does later in the fragment LH IV 7B, 4 Bl. 26. But in this case one is forced to look at things from an extensional point of view, since the subject is taken in a *part* of its extension.

[43] Of course, these subalternate moods are only as valid as subalternation itself, which modern logic regards as illegitimate. Leibniz thinks he can justify it by saying that if A includes B, then it doesn't exclude it, and that if A excludes B, it doesn't include it. But this supposes that A exists (that its extension is not empty). This is a noteworthy example of the illusions of language, which disguises such paralogisms with a false appearance of rigor and clarity.

[44] *Phil.*, VII, 210. In the second column we have put those moods obtained by retaining the *major* premise of the mood listed in the first column, and in the third those obtained by retaining the *minor*. To facilitate comparison, we have also subscripted the name of each mood with the number of the figure to which it belongs.

Barbara$_{\mathrm{I}}$	gives *Baroco*$_{\mathrm{II}}$	and *Bocardo*$_{\mathrm{III}}$[45]
Celarent$_{\mathrm{I}}$	— *Festino*$_{\mathrm{II}}$	— *Disamis*$_{\mathrm{III}}$
Cesare$_{\mathrm{II}}$	— *Ferio*$_{\mathrm{I}}$	— *Datisti*$_{\mathrm{III}}$
Camestres$_{\mathrm{II}}$	— *Darii*$_{\mathrm{I}}$	— *Ferison*$_{\mathrm{III}}$
Camenes$_{\mathrm{IV}}$	— *Dimaris*$_{\mathrm{IV}}$	— *Fresison*$_{\mathrm{IV}}$
Barbari$_{\mathrm{I}}$	gives *Camestros*$_{\mathrm{II}}$	and *Felapton*$_{\mathrm{III}}$
Celaront$_{\mathrm{I}}$	— *Cesaro*$_{\mathrm{II}}$	— *Darapti*$_{\mathrm{III}}$
Cesaro$_{II}$	— *Celaront*$_{\mathrm{I}}$	— *Darapti*$_{\mathrm{III}}$
Camestros$_{\mathrm{II}}$	— *Barbari*$_{\mathrm{I}}$	— *Felapton*$_{\mathrm{III}}$
Camenos$_{\mathrm{IV}}$	— *Baralip*$_{\mathrm{IV}}$	— *Fesapo*$_{\mathrm{IV}}$

As predicted, there are six moods represented twice each in this table,[46] namely the four subalternate moods, *Barbari* and *Camestros*, *Cesaro* and *Celaront*, where each member of a pair gives rise to the other, and then *Darapti* and *Felapton*, where each arises from the two preceding pairs. We therefore have just 24 distinct moods, which are the 24 valid moods already found by another method (the six from each figure).

10. This table also confirms Leibniz's thesis that the moods of the second and third figures can be reduced to those of the first through simple regression, whereas those of the fourth cannot be reduced without first undergoing conversion. Specifically, we should note that there is a reciprocity among the three moods on each line, namely, that by regression, any one of them can only give rise to the remaining two.[47] We can also see that in both halves of the table, the first four lines each contain one mood of the first figure, one mood of the second, and one mood of the third, while the fifth line is composed solely of moods from the fourth figure. By regression, then, we can pass from any one of the first three figures to another of them, but we can never escape from the fourth figure by this process alone.

The preceding table suggests several other remarks. First, *Barbara* gives rise to the only two moods beginning with a *B* (*Baroco* and *Bocardo*), which are deduced from it by *reductio ad absurdum*.[48] Further, the four universal moods beginning in *C* each give rise to one mood in *D* and one mood in *F*. The second half of the table does not exhibit this same symmetry. The four (principal) moods *Darapti*, *Felapton*, *Baralip* and *Fesapo* cannot be deduced from universal moods, but only from subalternate moods: *Darapti* from *Celaront* or *Cesaro*; *Felapton* from *Barbari* or *Camestros*; and *Baralip* and *Fesapo*, finally, from *Camenos*. There is therefore no connection between the two halves of the table, and we can see that in all the moods of the second half, a particular conclusion is deduced from universal premises. But this is a deduction that modern logic has shown to be invalid.[49] Moreover, the moods in the second half of the table are derived from those of the first by subalternation; but modern logic admits neither subalternation nor partial

[45] In the *New Essays*, IV.ii.1, Leibniz errs in calling *Disamis* the mood he deduces by regression from *Barbara*; in fact it is *Bocardo*.

[46] And more exactly, in the first four lines of the second half.

[47] See the table in §7.

[48] See Appendix I, §9.

[49] See n. 30.

conversion (which is known to be equivalent to it). We therefore cannot deduce the moods of the second half from those of the first. There remain then as valid moods only the first fifteen, which are those whose legitimacy modern logic has verified.[50]

If Leibniz had followed through the development of the main theme of his *Logical Definitions*,[51] namely that particular propositions serve only to deny the relations of inclusion and exclusion affirmed in universal propositions, he would have seen that one can no more deduce a particular proposition from two universals than a negative from two affirmatives;[52] then, instead of completing the list of classical moods with subalternate moods, he would on the contrary have excluded, along with these subalternate moods, the four moods of type UUP, which are equivalent to them and no more valid than them.[53]

11. Leibniz was very close to discovering the invalidity of these moods, for he at least recognized they were "imperfect."[54] The subalternate moods are imperfect, according to him, because they entail less than they could. That is, the moods *Barbari*, *Celaro*, *Cesaro*, *Camestros*, and *Camenos* have the same premises as *Barbara*, *Celarent*, *Cesare*, *Camestres*, and *Camenes* but derive from them a particular conclusion instead of a universal one.[55] On the other hand, the moods derived by regression from the subalternate moods (*Darapti*, *Felapton*, *Baralip*, and *Fesapo*) are imperfect because they suppose more than they have to. That is, these moods have universal premises where a particular would suffice to justify the same conclusion. We can see this by comparing *Darapti* to *Disamis* or *Datisi*, *Felapton* to *Bocardo* or *Ferison*, *Baralip* to *Dimaris*, and *Fesapo* to *Fresison*. These two imperfections come to the same thing, however, as Leibniz notes.[56] But the moods he declares to be imperfect because they have too strong a premise or too weak a conclusion are in fact invalid, for universal propositions, lacking the existential import of particular ones, are in this respect weaker than the corresponding particulars and not, as he thought, stronger.[57]

12. Let us now consider how Leibniz formulates the rules of the five universal moods from the point of view of comprehension:[58]

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[50] Note that the nine invalid moods compose the second horizontal row of the table extracted above (§4) from *On the Art of Combinations*. This is because that row corresponds to the quantitative mood UUP, which modern logic has shown to be invalid.

[51] Found in the unpublished fragment LH IV 7B, 4 Bl. 3 recto.

[52] Rule V (Appendix I).

[53] These four moods (*Darapti*, *Felapton*, *Baralip*, and *Fesapo*) are precisely those in which the letter *p* is found, i.e., those derived from moods of the first figure by means of partial conversion, which is, as we have seen, invalid.

[54] LH IV 7B, 4 Bl. 2 verso.

[55] For Leibniz, who allowed subalternation, the particular was weaker than the corresponding universal; we now know, however, that it implies something more, namely a judgment of existence.

[56] LH IV 7B, 4 Bl. 2 verso.

[57] The invalidity of the four moods *Darapti*, *Felapton*, *Baralip*, and *Fesapo* was first recognized by MacColl, who justified them through the addition of an existential judgment ("The Calculus of Equivalent Statements," *Proceedings of the London Mathematical Society*, vol. IX, 13 June 1878; cf. "Symbolic Reasoning," *Mind*, no. 17, January 1880). Whitehead has found to the contrary that these moods have premises which are too strong, precisely because he imputes to each universal proposition a judgment of existence (*Universal Algebra*, vol. I, p. 104, Cambridge, 1898).

[58] *Phil.*, VII, 209.

(1) "What includes an includer includes what is included," i.e, If A includes B, and if B includes C, then A includes C (*Barbara*).

(2) "What includes an excluder excludes what is excluded," i.e., If A includes B, and if B excludes C, then A excludes C (*Celarent*).

(3) "What includes an excluder is excluded from what is excluded," i.e., If A includes B, and if B excludes C, then C excludes A (*Camenes*). These are the same premises as in *Celarent*, but with the conclusion converted.[59]

(4) "What excludes what is included also excludes what includes it," i.e., If A excludes B, and if C includes B, then A excludes C (*Camestres*).

(5) "What excludes what is included is excluded from what includes it," i.e., If A excludes B, and if C includes B, then C excludes A (*Cesare*). These are the same premises as in *Camestres*, but with the conclusion converted.

As we can see, these five rules can be reduced to three, for the third and fifth derive from the second and fourth by simple conversion of the conclusion:

(1) If A includes B, and if B includes C, then A includes C.

(2) If A includes B, and if B excludes C, then A and C exclude each other.

(3) If A includes B, and if C excludes B, then A and C exclude each other.

These last two can even be reduced to a single rule, assuming that the relation of exclusion is reciprocal.[60] In that case we can simply say: If A includes B, and if B and C exclude each other, then A and C also exclude each other.

Leibniz himself summarizes all of these rules in the following two formulas:

I. A middle term (B) included in the subject (A) includes in the subject, or excludes from it, any predicate (C) that it includes or excludes.

II. A middle term (B) excluded from the subject (A) excludes from it any predicate (C) that it includes.[61]

The second formula, moreover, can be reduced to the first by a simple permutation of A and C, so we need only look at the first. This is composed of two parts, one of which is the principle of the single affirmative mood *Barbara*: "If A contains B, and if B contains C, then A contains C." This principle is obviously always valid, whether we interpret the containment in terms of extension of comprehension. The other half, however, which is the principle of the four negative moods, "If A contains B, and if B excludes C, then A excludes C," is in no way obvious;[62] it is true from the point of view of comprehension, but false from that of extension. It assumes that if one concept excludes another (from its comprehension), then it excludes it from all other concepts in which it is contained (in comprehension). In other words, two concepts that exclude each other also mutually exclude each other from the comprehension of a third, as if they could not coexist within it. Such an exclusion, however, does not hold from the extensional point of view: the

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[59] We can see from this that *Camenes* (of the fourth figure) is more closely related to *Celarent* (of the first) than are the universal moods of the second figure.

[60] If A excludes B, then B excludes A (*Phil*., VII, 208, no. 8).

[61] "1. When the middle term is included in the subject, a predicate that is included in (or excluded from) it is also included in (or excluded from) the subject.... 2. When the middle term is excluded from the subject, a predicate that is included in it is also excluded from the subject" (LH IV 7B, 4 Bl. 26). The same rules are found in *Phil*., VII, 209 (nos. 18, 19).

[62] Even though Leibniz dispenses with its demonstration by saying, "It is clear by itself" (*Phil*., VII, 209).

extensions of two mutually exclusive concepts can very well both comprise part of the extension of the same third concept.

13. This difference can be rendered visible in a geometric schematism commonly attributed to Euler[63] but already known to Leibniz.[64]

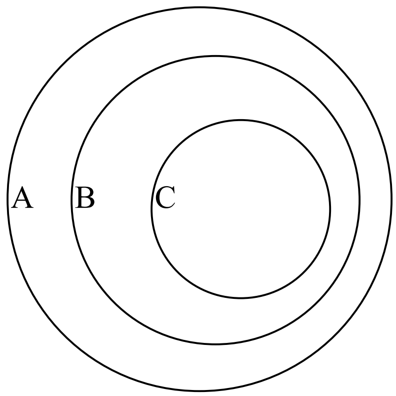

Fig. 1

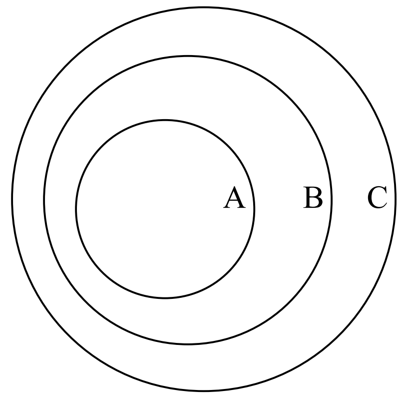

Fig. 2

We may represent the three terms of a syllogism by the same number of circles (interior when they are included, exterior when they are excluded). Now, clearly, the schema of *Barbara* remains the same whether we interpret the inclusion of concepts in terms of extension or comprehension, for we have only to invert the order of the terms (Figs. 1 and 2). But it is not the same with the schema of *Celarent* (which corresponds to the second part of the rule). From the point of view of extension, the minor term A, being contained in the middle term B, is excluded from the major term C, just like B itself, and this is revealed by a simple inspection of Figure 3. From the point of view of comprehension, by contrast, the middle term B, being contained in the minor term A, excludes from it the major term C which B itself excludes (Fig. 4).

[63] Since he employed it in his *Letters to a German Princess* (1768), letters 102-108.
[64] See *On the Verification of Logical Form by the Drawing of Lines* (LH IV 7B, 4 Bl. 1-14).

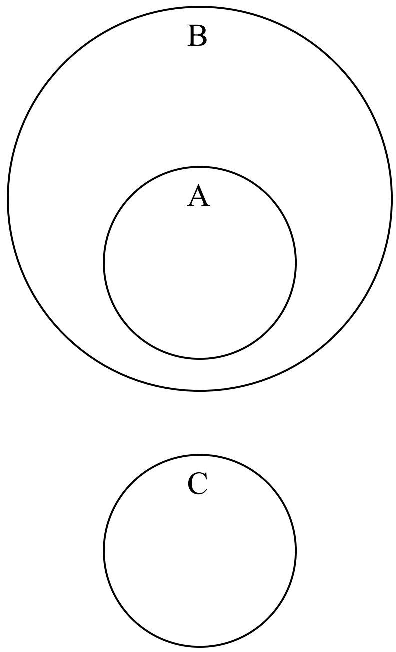

Fig. 3

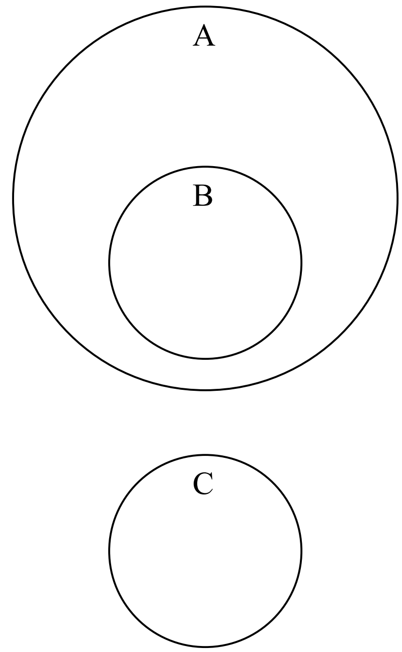

Fig. 4

This certainly is not at all obvious from the figure, and can only be understood by imagining a sort of repulsion between the terms B and C—more moral than physical—as if the presence of B in A banished C from this domain by forbidding it entry. This consideration of a dynamic participation of concepts is certainly legitimate, though rather vague and obscure. It can hardly assist and guide our reasoning but would sooner entangle it and lead it astray. In any case, it is easily seen to be resistant to a geometrical schematism and hence to a mathematical treatment, which is always based on some intuition.[65]

14. Nevertheless, Leibniz appears to have constantly oscillated between the two opposing points of view of extension and comprehension, and never to have definitively opted for one over the other. At first he seems to prefer that of comprehension, and to consider "ideas" rather than "individuals," contrary to the custom of the schools.[66] Later, however, he frees himself from the authority of Aristotle, imputing to him in 1686 the doctrine according to which the predicate is contained in the subject (of the universal affirmative).[67] He admits that one considers the species as forming part of the genus and notes that in order to obtain geometrical schemata conforming to the point of view of extension, it suffices to invert those arrived at from the point of view of comprehension, and indeed only the schemata of UA and PN propositions, since those of UN and PA

[65] This is demonstrated by algorithmic logic, which could be established only after concepts had been reduced to their extensions, i.e., to *sets* or *classes* of objects or individuals.

[66] For example in the logical essays of April 1679 (LH IV 5, 8b, §12; LH IV 5, 8f).

[67] *General Investigations*, §§16 and 132 (LH IV 7C Bl. 22 verso, 29 recto). Cf. *Example of an Enlightening Geometry* (*Math*., VII, 261); see Chap. 7, §9.

propositions are symmetrical and thus do not change.[68] In 1690, he distinguishes these two contrary methods very clearly and seems to think they are equivalent;[69] the next day, however, he implicitly regards the subject as containing the predicate.[70] Just once does he reveal a decided preference for the extensional, in the (unfortunately undated) study entitled *A Mathematics of Reason*, throughout which he considers the individuals that compose the extensions of various terms and bases all the syllogistic rules on this consideration alone.[71] In short, Leibniz seems to have remained undecided between these two points of view, while showing a marked predilection, albeit mechanical and almost instinctive, for that of comprehension, constantly reiterating, as if from habit, that the predicate is contained in the subject.[72]

15. This tendency is even less comprehensible when we consider that Leibniz had all sorts of reasons to prefer the extensional point of view, the only one consistent with the principles of his logic. As we have just remarked, it is from the extensional point of view that he succeeds in formulating and justifying the rules of the syllogism in the most precise and complete way.[73] Nor could he have succeeded by any other method. He wanted to base these rules on the rule of distributed and non-distributed terms (i.e. general and particular terms), but this rule rests on the quantification of the predicate by virtue of its form, and so on the consideration of extension. To say that the predicate is universal in negative propositions and particular in affirmative propositions is also to say that it is to be taken in the totality or in a part, respectively, of its extension. And Leibniz repeatedly endorses this rule, which serves, as he himself says, as the foundation of all the rules governing the figures and moods of the syllogism.[74] We can notice, too, that

[68] Ibid., §§122, 123 (LH IV 7C Bl. 28 recto).

[69] Study of 1 August, 1690 (LH IV 7B, 2 Bl. 3 recto).

[70] *Foundations of the Logical Calculus*, 2 August 1690, §17 (LH IV 7C Bl. 97).

[71] LH IV 6 Bl. 14; cf. LH IV 7B, 4 Bl. 7.

[72] For example, LH IV 5, 8b; LH IV 7B, 4 Bl. 15; LH IV 7C Bl. 73-74, 103. Elsewhere Leibniz expressly prefers comprehension to extension (LH IV 7B, 2 Bl. 70-74).

[73] *A Mathematics of Reason* (LH IV 6, 14).

[74] "I should like to know who first came up with the observation concerning distributed and non-distributed terms, whence follows the rule that a term that is not distributed in the premises cannot be distributed in the conclusion; for there is no hint at all of this in Aristotle, and yet the useful moods can be demonstrated more easily from these rules than from Aristotle's." Leibniz to Koch, 2 September 1708 (*Phil*., VII, 478).

"I should also like to know who first came up with the doctrine that derives the quantity of the predicates from the quality of propositions, and which shows that every predicate of a negative proposition is universal and that every predicate of an affirmative proposition is (by virtue of its form) particular—a consideration that was already known to some scholastics and which provides a remarkably short demonstration of the moods, but which, unless I am wrong, is nowhere to be found in Aristotle." Letter to Koch, 31 August 1710 (*Phil*., VII, 481).

"(1) The middle term must be universal in at least one premise.... (2) At least one premise must be affirmative.... (3) A term that is particular in a premise must be particular in the conclusion.... (4) The subject of a universal proposition is universal, that of a particular proposition is particular.... (5) The predicate of an affirmative proposition is, by virtue of its form, particular, that of a negative is universal.... From these five principles, all the theorems concerning the figures and moods can be demonstrated." *A Mathematics of Reason* (LH IV 6, 14 Bl. 4 verso).

Finally, in the opuscule *On the Verification of Logical Form by the Drawing of Lines*, we read: "From this, the discovery of distributed and non-distributed terms becomes clear.... a *distributed term* is the same as a total or universal term; a *non-distributed term* is particular or partial. The quantity of the subject even though he expressly argues against *explicit* quantification of the predicate (just as Hamilton was later to propose), this is precisely because the predicate is already implicitly quantified in virtue of its form, according to whether the proposition is affirmative or negative.[75]

16. At the same time, Leibniz tried to represent arguments, especially syllogistic arguments, by geometrical figures and attached a great importance to this schematism, as we shall see when we come to study his characteristic (Chap. 4). Not only did he precede Euler in the invention of circular schemata for all the moods of the syllogism, but he also invented a system of linear schemata that is even more ingenious and complete. We shall proceed to sketch this out, following his unpublished manuscripts.[76]

The linear schematism he employs in the logical studies that have already been published is well known.[77] Essentially, it consists in representing concepts by segments of a single straight line that are included in one another, or outside one another, or have a part in common, etc. To distinguish these different segments, however, and also to show their unity, we must either use brackets of some sort or else designate them by their endpoints, which complicates the symbolism. Leibniz perfects the system by separating the segments that previously were run together on one line and arranging them parallel to one another, in such a way that it suffices to draw perpendicular (dotted) lines from their endpoints to indicate their relations of inclusion or exclusion, either partial or total.

Thus the four classical propositions will be represented by the following schemata:

UA All B is C }

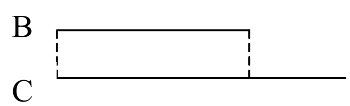

Fig. 5

UN No B is C}

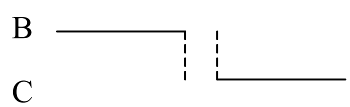

Fig. 6

is the same as that of the proposition.... But the predicate in every affirmative proposition is particular (or non-distributed) and is total (or distributed) in every negative proposition, since the subject is either wholly or in part excluded from the entire predicate. From this consideration of distributed and non-distributed terms follow the well-known theorems and rules that determine the general laws for the figures and moods" (LH IV 7B, 4 Bl. 7 recto; cf. 1 recto and 9). Leibniz again expresses the desire, as in the letters to Koch, to know who discovered this principle. (See the application and justification of these views in Appendix I.)

[75] *A Mathematics of Reason* (LH IV 6, 14 Bl. 1 recto). Cf. *Elements of a Universal Calculus*, April 1679: "Every man is a rational thing" (LH IV 5, 8c Bl. 13-16).

[76] LH IV 6 Bl. 15; LH IV 7B, 2 Bl. 18; LH IV 7B, 4 Bl. 1-14; LH IV 7C Bl. 28.

[77] Namely, in fragments XIX and XX of Gerhardt (*Phil.*, VII, 228-247). See Chap. 8, §20.

PA Some B is C}

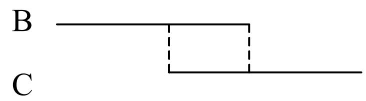

Fig. 7

PN Some B is not C}

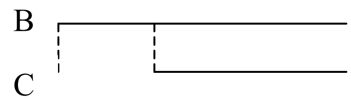

Fig. 8

Leibniz takes certain precautions in drawing these schemata, in order not to have them say more than the corresponding propositions. He notes that the schemata for the UN and the PA are symmetrical, since these propositions are simply convertible. He is careful though to make those of the UA and the PN asymmetrical, as much to distinguish them from the others as to prevent anyone from taking them to be simply convertible from an inspection of the figure.

These schemata have an advantage over the circular schemata (named after Euler) that Leibniz does not mention but of which he was no doubt aware when he juxtaposed both kinds in the margins of his sketch, namely that, thanks to the dotted lines, they allow us to distinguish the two types of particulars, affirmative and negative. The circular schema in Fig. 9 does not permit this, for it could be taken to represent indifferently the propositions “Some B is C” and “Some B is not C.”

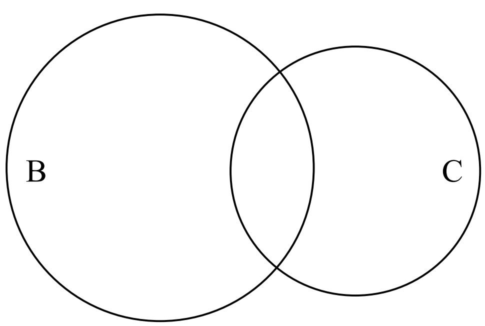

Fig. 9

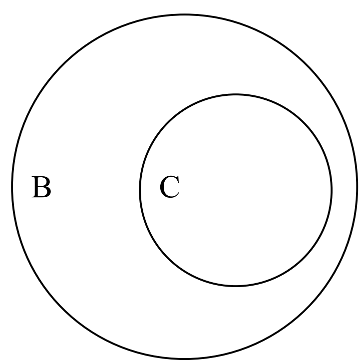

Fig. 10

Nor is this accomplished by the circular schema in Fig. 10, which can represent both “Some B is not C” and “All C is B.” The dotted lines display the exact sense of the proposition and delimit in each term the affected part. The proposition is affirmative when the dotted lines determine real segments for each term, and negative when they pass by and outside at least one of them and fall, so to speak, into the void.

This was, in fact, so much Leibniz's intention in inventing these schemata that he was led to make this even more explicit by doubling the part of each line that carries the affirmation or negation. In this way he obtains the following schemata:[78]

UA

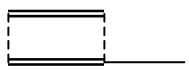

Fig. 11

UN

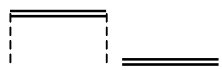

Fig. 12

PA

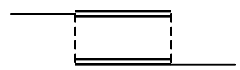

Fig. 13

PN

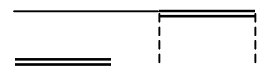

Fig. 14

The doubled segment represents the *affected* part of each term, i.e., the part that is expressly included in or excluded from the other term. It follows from this that a term is taken universally (or distributed) when its line is completely doubled, and taken particularly (or non-distributed) when its line is doubled only in part. Thus we see that the subject of a universal proposition is universal, that of a particular proposition is particular; the predicate of an affirmative is particular, that of a negative is universal; and finally that the UA and PN possess one distributed term, the UN two and the PA none at all. In short, we discover intuitively all the rules from which Leibniz aims to deduce the valid moods of the syllogism. Let us note clearly that this arises from holding consistently to the extensional point of view and by considering, for example, the subject of the UA as *contained* in the predicate.

17. To form the schema of a syllogism, Leibniz simply juxtaposes the schemata of the two premises by placing in the center those segments that represent the middle term in each of them, and by making them coincide. In this way he obtains a schema with three lines, the outermost of which represent the extreme terms. He then joins these with two vertical solid lines representing the conclusion (to distinguish them from the dotted lines that represent the premises). The segments marked off by the conclusion (i.e. those included between these two vertical solid lines) are necessarily contained in those marked off by the premises (those included in the dotted lines). Leibniz underlines or doubles the affected part of the minor term in order to indicate whether the conclusion is universal or particular (according to whether the minor term is doubled in whole or only in part). The schemata of the moods of the first figure, by way of example, are these:

Barbara

A	All C is B	B
A	All D is C	C
A	All D is B	D

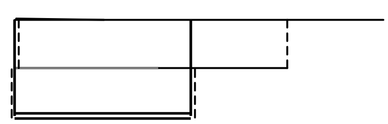

Fig. 15

[78] LH IV 7B, 4 Bl. 7.

Celarent

E	No C is not B	B
A	All D is C	C
E	No D is not B	D

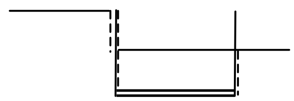

Fig. 16

Darii

A	All C is B	B
I	Some D is C	C
I	Some D is C	D

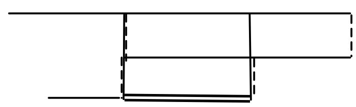

Fig. 17

Ferio

E	No C is not B	B
I	Some D is C	C
O	Some D is not B	D

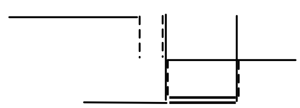

Fig. 18

We can see how this schematism is more explicit and more expressive than the circles of so-called Euler diagrams.[79] Leibniz also employs it to verify whether a proposed mood is legitimate or whether a given syllogism is valid. Suppose, for example, that we would like to know whether the fourth figure admits an AOO mood. We would construct it as follows:

[79] This schematism appears, still in a rather crude form, in *A Mathematics of Reason* (LH IV 6, 14 Bl. 4 recto), and then in the fragment LH IV 7B, 2 Bl. 18, where Leibniz does not yet employ intersecting lines. Instead he indicates the relations of the terms by limiting the corresponding segments, or by marking off with dots their possible extensions. He represents *Barbara*, for instance, as follows

A.CD	B
A.BC	C
A.BD	D

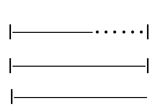

Fig. 20

which signifies that B, contained in the middle term C, can at most be equal to it, and that D, which is at least equal to C, can be further extended (to the right). Leibniz appreciates the need to distinguish universal from particular terms more clearly in this diagram. In general, a universal term is limited on both sides, while a particular term is unlimited. It is in the *General Investigations* of 1686 that the doubling of lines designed to mark off the affected part of each term first appears. This schematism is found until the end of Leibniz's life, in a fragment dating from 1715 or 1716 (LH IV 6, 15 Bl. 7 and 9).

A | All B is C | B
O | Some C is not D | C
O | Some D is not B | D

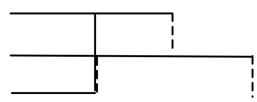

Fig. 19

The diagram shows that even if the premises are true, the conclusion could be false and does not therefore follow from them.

18. It is rather curious that after having applied this schematism successfully to the valid syllogistic moods considered *from the point of view of extension*, Leibniz then tries in the same study to apply it *from the point of view comprehension*. For example, he represents the UA by the schema:

Every man is an animal | B
Every B is C | C

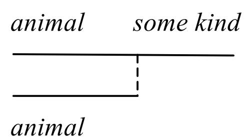

Fig. 21

which he explains in this way: "A man is the same as some kind of animal," namely a rational one. Here we see clearly that he considers the respective comprehensions of two terms, since he regards *animal* as being contained in *man*, and *man* as equal to *animal* plus some other attribute (*rational*) that completes its comprehension. But he does not succeed in translating even the four classical propositions in this way, let alone the syllogistic moods, and he seems to have renounced the attempt to do so. This failure is significant and shows that only the point of view of extension is consistent with this geometrical schematism.

In the *General Investigations* of 1686, Leibniz takes the inverse approach. He begins by defining the schemata of A, E, I, O from the point of view of comprehension (§§113-121).[80] Thus the UA "A is B" is translated by the schema:

A
B

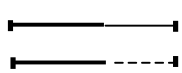

Fig. 22

The UN and the PA are translated by the symmetrical schemata:

A
B

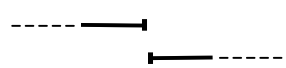

Fig. 23

[80] LH IV 7C Bl. 28 recto

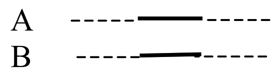

Fig. 24

Finally, the PN is translated by the schema:[81]

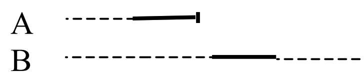

Fig. 25

which clearly indicates that A does not contain B (in comprehension), just as that of the UA indicates that A does contain B (in comprehension).

Having done this, Leibniz remarks that, inversely, one can consider a species as part of a genus (from the point of view of extension) and that from this point of view, the schema for the UA is the inverse of the preceding one, namely:

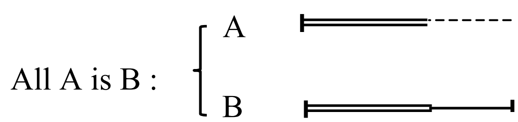

Fig. 26

It is the same with that of the PN, he says; and, indeed, it suffices to permute the letters A and B in Fig. 25 in order to obtain the translation (in extension) of the proposition "Some A is not B." With regard to the schemata of the UN and PA, these are not changed by the permutation, for they are both symmetrical. Leibniz believes that he can thus conclude that the new schemata (in extension) do not differ from the former ones (in comprehension) except by inversion of the lines (§123).

19. Even if this is true for propositions containing just two terms, it is not the case for the schemata of syllogisms, which contain three terms. Certainly the schema of *Barbara* is no different whether interpreted in terms of extension or comprehension, for we have only to permute the outermost terms. But the schema of *Celarent*, for example, is no longer valid even granting this permutation. Let us construct it from the point of view of comprehension:

E	No C is not D
A	All B is C
E	No B is not D

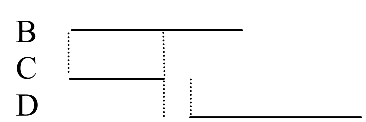

Fig. 27

[81] Note that Leibniz doubles the affected parts (as in LH IV 7B, 4 Bl. 7) and also uses bounded and dotted segments (as in LH IV 7B, 2 Bl. 18).

We see that when the premises are true, the conclusion "Some B is D" is possible in virtue of the figure, and so the (contradictory) conclusion "No B is D" does not follow from the figure. This proves once again that relations of comprehension are not susceptible to a geometrical representation like relations of extension, and that it is not sufficient merely to reverse or invert the latter relations to obtain the former. Leibniz was therefore mistaken in thinking that the former are purely and simply inverses of the latter, and we shall see that this error plagued his attempts at a logical calculus and contributed to their abandonment (Chap. 8).

In any event, Leibniz was constantly attracted by both of these contrary tendencies, the one deriving from tradition and leading him to give precedence to relations of comprehension, the other more in accord with his mathematical mind and leading him oftentimes to prefer an extensional approach.[82] But the latter is the only one that allows us to give a mathematical treatment of logic, since, as we have seen, it is the only one that fulfills the conditions of intuition and imagination.[83]

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[82] This, for example, is what leads him to prefer to arrange the premises of a syllogism in the order suggested by Ramus (the minor first), which corresponds to a consideration of extension, rather than in the order of Aristotle (the major first) which derives from a consideration of comprehension: "I am of your opinion... that this other arrangement is better: All A is B, all B is C, therefore all A is C" (*New Essays*, IV.xvii.4; cf. §8: "The usual mode of expression has more to do with individuals, while that of Aristotle considers ideas or universals").

[83] As we shall see, mathematics is for Leibniz the logic of the imagination (*New Elements of Universal Mathematics*, LH IV 7B, 6 Bl. 9; cf. LH XXXV 1, 26a).